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Soft supersymmetry breaking in deformed moduli spaces, conformal theories and $\mathcal{N} = 2$ Yang-Mills theory

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Abstract: We give a self-contained discussion of recent progress in computing the non-perturbative effects of small non-holomorphic soft supersymmetry breaking, including a simple new derivation of these results based on an anomaly-free gauged $\mathrm{U}(1)_R$ background. We apply these results to $\mathcal{N} = 1$ theories with deformed moduli spaces and conformal fixed points. In an SU(2) theory with a deformed moduli space, we completely determine the vacuum expectation values and induced soft masses. We then consider the most general soft breaking of supersymmetry in $\mathcal{N} = 2$ SU(2) super-Yang-Mills theory. An $\mathcal{N} = 2$ superfield spurion analysis is used to give an elementary derivation of the relation between the modulus and the prepotential in the effective theory. This analysis also allows us to determine the non-perturbative effects of all soft terms except a non-holomorphic scalar mass, away from the monopole points. We then use an $\mathcal{N} = 1$ spurion analysis to determine the effects of the most general soft breaking, and also analyze the monopole points. We show that naïve dimensional analysis works perfectly. Also, a soft mass for the scalar in this theory forces the theory into a free Coulomb phase.

Keywords: Supersymmetry Breaking, Supersymmetry and Duality, Extended Supersymm etry, Supersymmetric Effective Theories.

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1. Introduction

In the last several years there has been significant progress in understanding the low-energy dynamics of strongly-coupled supersymmetric gauge theories [1, 2, 3]. Most of this progress has been limited to holomorphic quantities, which give a great deal of interesting information if supersymmetry (SUSY) is exact. In many cases, the moduli space of vacua and the phase structure and massless excitations of the theory can be exactly determined. A natural question to ask is whether these results can be extended to the case of explicit breaking of SUSY. As a first step, one can study the case where SUSY is broken softly by mass parameters that are small compared to the scale of strong dynamics in the gauge theory. In cases where the low-energy effective field theory is known in the SUSY limit, one can carry out an analog of chiral perturbation theory for SUSY breaking.

The most general soft SUSY breaking can be parameterized by turning on higher θ-dependent terms in the coupling constants viewed as superfield spurions [4]. For example, if we write

$$\mathcal{L} = \int d^4\theta\, \mathcal{Z} Q^\dagger e^V Q + \left(\int d^2\theta\, S\, \mathrm{tr}(W^\alpha W_\alpha) + \text{h.c.} \right) + \cdots \tag{1.1}$$

with

$$S = \frac{1}{2g^2} + \theta^2 \frac{m_\lambda}{g^2}, \qquad \mathcal{Z} = Z \left[1 - \theta^2 \bar{\theta}^2 m^2\right]. \tag{1.2}$$

then m_λ is a gaugino mass and m^2 is a scalar mass. The effects of soft SUSY breaking that can be parameterized by chiral superfields can be studied using holomorphy and SUSY non-renormalization theorems [5, 6, 7]. However, when studying the non-perturbative effects of soft SUSY breaking, non-holomorphic scalar masses cannot be neglected compared to holomorphic soft terms such as gaugino masses. (For example, in an asymptotically free theory, if scalar masses are smaller than gaugino masses at a renormalization scale where the theory is weakly coupled, then the renormalization group will generate a scalar mass comparable to the gaugino mass at the scale where the theory becomes strongly coupled.) In superfield language, the problem is therefore to determine how the superfield $\mathcal{Z}$ in the fundamental theory couples to fields in the low-energy theory. Ref. [8] pointed out that one obtains nontrivial information by viewing $\mathcal{Z}$ as a gauge superfield. The point is that $\mathcal{Z}$ contains a vector field

$$\mathcal{Z} = \cdots + \theta \sigma^\mu \bar{\theta} A_\mu + \cdots \tag{1.3}$$

that couples to the Noether current associated with a U(1) 'Q number' symmetry. As is well-known, this means that the dependence on A_μ at low energies are controlled simply by charge conservation. SUSY relates this to the dependence on the soft mass, and one obtains non-perturbative information about non-holomorphic SUSY breaking at low energies.

To make this idea precise, one must deal with several technical complications. First, one must understand the renormalization properties of the superfield couplings [11, 12, 13]. Second, U(1) 'gauge' symmetries such as the one discussed above are generally anomalous. This does not give rise to any inconsistency (the relevant gauge fields are non-dynamical sources), but it does mean that the U(1) symmetry is broken explicitly, and this must be properly taken into account. These problems were addressed in ref. [8] in a 'wilsonian' language, and used to obtain results in several theories of interest.

In the present paper, we extend the results of ref. [8] in several ways. First, we give a self-contained review of the method of ref. [8] in terms of renormalized couplings and superfield RG invariants. We also give a new derivation based on a non-anomalous gauged $\mathrm{U}(1)_R$ symmetry in a supergravity background. We apply these results to several classes of $\mathcal{N} = 1$ theories that were not treated in ref. [8], namely those with deformed moduli spaces and conformal fixed points. In the SU(2) theory with a deformed moduli space, we are able to determine the vacuum uniquely for vanishing gaugino masses, and compute the soft masses of the composite fields for arbitrary perturbations at the maximally symmetric point. In the conformal window of SUSY QCD, we give a very simple derivation of the fact that soft masses scale to zero as one approaches the fixed point. This result was previously obtained in ref. [9] by explicit calculation.

We then turn to $\mathcal{N} = 2$ SU(2) super-Yang-Mills theory. This was studied in ref. [7] for a special subset of the possible soft SUSY breaking terms. We generalize these results to include the non-perturbative effects of the most general possible soft SUSY breaking. We first perform a spurion analysis in terms of $\mathcal{N} = 2$ superfields that includes all soft breaking terms except a non-holomorphic scalar mass. This analysis also leads to an elementary derivation of the relation between the modulus and the effective prepotential that had previously been obtained using properties of the Seiberg-Witten solution. We then analyze the theory using the $\mathcal{N} = 1$ techniques discussed above. In this way, we are able to determine the exact potential on the full moduli space for general soft SUSY breaking, including the potential near the monopole points. The agreement between the two calculations serves as a nontrivial check on our methods.

We find a rich structure of phase transitions in this theory as a function of the soft masses. For example, when fermion masses dominate, the vacuum is near the monopole/dyon points and exhibits confinement via monopole/dyon condensation; when the scalar mass dominates, the vacuum is at the origin and the theory is in a Coulomb phase. We also show that 'naïve dimensional analysis' works perfectly for all the quantities we compute, giving strong support for these methods in strongly coupled SUSY theories.

2. Non-perturbative non-holomporphic $\mathcal{N} = 1$ soft SUSY breaking

In this section, we review the results of ref. [8] on the non-perturbative effects of soft $\mathcal{N} = 1$ SUSY breaking, including non-holomorphic scalar masses. Our discussion here uses renormalized couplings rather than the 'wilsonian' language of ref. [8], but all results are completely equivalent. We then apply this formalism to the case of deformed moduli spaces. This case has not been considered before, so the results are interesting in their own right. This case also has some important similarities with the $\mathcal{N} = 2$ super-Yang-Mills theory we will study in the following section.

2.1 RG invariant superfield spurions

Consider a supersymmetric gauge theory defined in the ultraviolet by a renormalized lagrangean at a scale μ where the theory is weakly coupled. We consider here only the case where the gauge group is simple and there are no superpotential terms. The renormalized lagrangean in superspace is

$$\mathcal{L}(\mu) = \int d^4\theta \sum_r \mathcal{Z}_r(\mu)\Phi_r^\dagger e^{V^{(r)}}\Phi_r + \left(\int d^2\theta\, S(\mu)\,\mathrm{tr}(W^\alpha W_\alpha) + \text{h.c.}\right). \qquad (2.1)$$

The renormalized couplings $\mathcal{Z}_r(\mu)$ and $S(\mu)$ can be promoted to superfields to all orders in perturbation theory [13], and SUSY breaking can be included by non-zero

θ^2 and $\bar\theta^2$ dependence in the couplings:

$$\begin{aligned}\mathcal{Z}_r(\mu) &= Z_r(\mu)\left[1-\theta^2 B_r(\mu)-\bar\theta^2 B_r^\dagger(\mu)-\theta^2\bar\theta^2\left(m_r^2(\mu)-|B_r(\mu)|^2\right)\right],\\ S(\mu) &= \frac{1}{2g_S^2(\mu)}-\frac{i\Theta}{16\pi^2}+\theta^2\frac{m_{\lambda S}(\mu)}{g_S^2(\mu)}\,. \end{aligned}\tag{2.2}$$

The quantity $\mathcal{Z}_r$ is a real superfield. Its components are defined so that Z_r is the usual wavefunction factor, B_r is a B-term. An elementary but important point is that the θ^2 terms affect the equation of motion for the auxiliary fields, with the result that the physical soft mass depends on the logarithm of the superfield $\mathcal{Z}_r$:

$$m_r^2 = -[\ln \mathcal{Z}_r]_{\theta^2\bar\theta^2}\,. \tag{2.3}$$

The quantity S is chiral and runs only at one loop [14]. Its components are defined so that Θ is the vacuum angle, and at 1-loop level, g_S is the gauge coupling and $m_{\lambda S}$ is the gaugino mass. However, g_S and $m_{\lambda S}$ differ from the conventionally-defined renormalized gauge coupling g and gaugino mass m_λ at two loops and beyond [14]. One manifestation of this is the fact that under the transformation

$$\begin{aligned}\Phi_r &\mapsto e^{A_r}\Phi_r\,,\\ \mathcal{Z}_r(\mu) &\mapsto e^{-(A_r+A_r^\dagger)}\mathcal{Z}_r(\mu)\,,\end{aligned}\tag{2.4}$$

where A_r is a constant chiral superfield, the coupling S has an anomalous transformation

$$S(\mu)\mapsto S(\mu)-\sum_r \frac{t_r}{8\pi^2}A_r\,. \tag{2.5}$$

Here, t_r denotes the index of the representation r.[1] For A_r pure imaginary, eq. (2.4) is a $\mathrm{U}(1)\times\cdots\times\mathrm{U}(1)$ transformation with charges

$$\mathcal{Q}_r(\Phi_s) = +\delta_{rs}\,, \tag{2.6}$$

and eq. (2.5) is a manifestation of the chiral anomaly. For A_r pure real, eq. (2.4) is a rescaling of the fields under which physical quantities are invariant, and eq. (2.5) is a manifestation of the Konishi (field rescaling) anomaly [15]. Note that $\mathcal{Z}_r$ transforms as a U(1) gauge superfield. The non-perturbative validity of these 'anomalous U(1)' symmetries is the crucial new ingredient introduced in ref. [8] to analyze the non-perturbative effects of soft masses in the theory.

The couplings g and m_λ are the lowest components of a *real* superfield

$$\begin{aligned}R(\mu) &= \frac{1}{g^2(\mu)}+\left(\theta^2\frac{m_\lambda(\mu)}{g^2(\mu)}+\text{h.c.}\right)+\cdots\\ &= S(\mu)+S^\dagger(\mu)+\frac{t_G}{8\pi^2}\ln R(\mu)-\sum_r\frac{t_r}{8\pi^2}\ln\mathcal{Z}_r(\mu)+\mathcal{O}(R^{-1})\,.\end{aligned}\tag{2.7}$$

[1] The index t_r is normalized to $\frac{1}{2}$ for fundamentals.

R is invariant under the transformation eq. (2.4). One can choose a special 'NSVZ' scheme in which all of the $\mathcal{O}(R^{-1})$ and higher corrections vanish, but this will not be important for our results. For a discussion of the superfield R (and in particular the role of its $\theta^2\bar{\theta}^2$ component) see ref. [13].

If this theory is asymptotically free, the strong dynamics occurs at a scale $\mu \sim \Lambda$ where the gauge coupling becomes large. The scale Λ must clearly be RG invariant. With the ingredients above, we see that we can form *two* RG-invariant scales:

$$\Lambda_S \equiv \mu \exp\left\{-\frac{16\pi^2 S(\mu)}{b}\right\}, \qquad \Lambda_R \equiv \mu \exp\left\{-\int^{R(\mu)} \frac{dR}{\beta(R)}\right\}, \tag{2.8}$$

where

$$b = 3t_G - \sum_r t_r\,, \qquad \beta(R) \equiv \mu\frac{dR}{d\mu}\,. \tag{2.9}$$

The RG invariant scale Λ_S is a chiral superfield, and transforms under eq. (2.4) as

$$\Lambda_S \mapsto \left(\prod_r e^{2t_r A_r/b}\right)\Lambda_S\,. \tag{2.10}$$

In other words, Λ_S is charged under the anomalous U(1):

$$\mathcal{Q}_r(\Lambda_S) = \frac{2t_r}{b}\,. \tag{2.11}$$

Because Λ_S is chiral, it is the scale that appears in non-perturbatively generated effective superpotentials. The transformation property eq. (2.10) is exactly what is required to make the effective superpotential invariant under the anomalous U(1). For example, in a theory with a simple gauge group and vanishing superpotential, the *anomaly-free* symmetries constrain the dynamically-generated superpotential to have the form [16]

$$W_{\text{eff}} \sim \frac{1}{\Lambda_S^{b/(t-t_G)}}\prod_r \Phi_r^{2t_r/(t-t_G)}\,, \tag{2.12}$$

where $t = \sum_r t_r$ is the total matter index, and the factors of Λ_S have been inserted by dimensional analysis. One can now check that the power of Λ_S is precisely what is required in order for W_{eff} to be invariant under the transformation eq. (2.4).

The RG-invariant scale Λ_R in eq. (2.8) is a real superfield defined by analytically continuation into superspace. Specifically, for real $R(\mu)$ eq. (2.8) defines a function of $R(\mu)$ (up to a multiplicative constant), and the continuation into superspace is defined by evaluating this function for $R(\mu)$ replaced by a real superfield. Λ_R is invariant under the transformation eq. (2.4).

Another important RG invariant is the quantity

$$\hat{\mathcal{Z}}_r \equiv \mathcal{Z}_r(\mu)\exp\left\{-\int^{R(\mu)} dR\,\frac{\gamma_r(R)}{\beta(R)}\right\}, \tag{2.13}$$

where

$$\gamma_r(R) \equiv \mu \frac{d \ln \mathcal{Z}_r}{d\mu}\,. \tag{2.14}$$

$\hat{\mathcal{Z}}_r$ can be thought of as the wavefunction factor for the field Φ_r renormalized at the RG-invariant superfield scale Λ_R. Like Λ_R, the quantity $\hat{\mathcal{Z}}_r$ is defined as a superfield by analytic continuation. Although $\hat{\mathcal{Z}}_r$ is RG invariant, it is not physical by itself because it transforms under field rescalings like a gauge superfield (like $\mathcal{Z}_r(\mu)$):

$$\hat{\mathcal{Z}}_r \mapsto e^{-(A_r + A_r^\dagger)}\hat{\mathcal{Z}}_r\,. \tag{2.15}$$

However, $\hat{\mathcal{Z}}_r$ can appear in the effective lagrangean: its transformation property is just what is required to write kinetic terms invariant under the transformation eq. (2.4).

Following ref. [8], we can also define the RG-invariant

$$I \equiv \Lambda_S^\dagger \left(\prod_r \hat{\mathcal{Z}}_r^{2t_r/b} \right) \Lambda_S\,, \tag{2.16}$$

which is also invariant under the anomalous U(1)'s. It is easily shown that

$$I = \text{constant} \times \Lambda_R^2\,, \tag{2.17}$$

so this does not give an independent invariant.

When the theory includes explicit SUSY breaking, the RG invariants above have θ-dependent components:

$$[\ln \Lambda_S]_{\theta^2} = -\frac{16\pi^2}{b}\frac{m_{\lambda S}}{g_S^2}\,, \tag{2.18}$$

$$[\ln \Lambda_R]_{\theta^2} = -\frac{1}{\beta(R)}[R]_\theta^2 = -\frac{8\pi^2}{b}\frac{m_\lambda}{g^2} + \cdots\,, \tag{2.19}$$

$$[\ln \hat{\mathcal{Z}}_r]_{\theta^2} = -B_r - \frac{\gamma_r(R)}{\beta(R)}[R]_{\theta^2} = -B_r - \frac{2C_r}{b} m_\lambda + \cdots\,, \tag{2.20}$$

$$\begin{aligned} [\ln \Lambda_R]_{\theta^2\bar\theta^2} &= -\frac{1}{\beta(R)}[R]_{\theta^2\bar\theta^2} + \frac{\beta'(R)}{\beta^2(R)}[R]_{\theta^2}[R]_{\bar\theta^2} \\ &= -\frac{1}{b}\sum_r t_r \left(m_r^2 - \frac{2C_r}{b}|m_\lambda|^2 \right) + \cdots\,, \end{aligned} \tag{2.21}$$

$$\begin{aligned} [\ln \hat{\mathcal{Z}}_r]_{\theta^2\bar\theta^2} &= -m_r^2 - \frac{\gamma_r(R)}{\beta(R)}[R]_{\theta^2\bar\theta^2} - \frac{d}{dR}\left(\frac{\gamma_r(R)}{\beta(R)} \right)[R]_{\theta^2}[R]_{\bar\theta^2} \\ &= -m_r^2 + \frac{2C_r}{b}|m_\lambda|^2 + \cdots\,. \end{aligned} \tag{2.22}$$

The ellipses denote terms that are suppressed at weak coupling; they can be computed exactly in the NSVZ scheme, but this is not important for our results.

The quantities in eqs. (2.18)–(2.21) are RG invariants by construction, and can therefore be evaluated at any value of the renormalization scale μ. In an asymptotically free theory, they simplify if they are evaluated in the limit $\mu \to \infty$:

$$[\ln \Lambda_R]_{\theta^2} = -\frac{8\pi^2}{b}\frac{m_{\lambda 0}}{g_0^2}\,, \tag{2.23}$$

$$[\ln \hat{\mathcal{Z}}_r]_{\theta^2} = -B_{r0}\,, \tag{2.24}$$

$$[\ln \Lambda_R]_{\theta^2\bar\theta^2} = -\frac{1}{b}\sum_r t_r m_{r0}^2\,, \tag{2.25}$$

$$[\ln \hat{\mathcal{Z}}_r]_{\theta^2\bar\theta^2} = -m_{r0}^2\,, \tag{2.26}$$

where

$$\frac{m_{\lambda 0}}{g_0^2} \equiv \lim_{\mu\to\infty}\frac{m_\lambda(\mu)}{g^2(\mu)}\,, \qquad B_{r0} \equiv \lim_{\mu\to\infty} B_r(\mu)\,, \qquad m_{r0}^2 \equiv \lim_{\mu\to\infty} m_r^2(\mu)\,. \tag{2.27}$$

We can make a field redefinition to set $B_{r0} = 0$; since we are considering the case where there is no superpotential, this has no further effect. We see that the SUSY breaking components of the RG-invariant superfield spurions are simple combinations of the *bare* coupling constants. This interpretation emerges very directly in the 'wilsonian' approach of ref. [8]. It is interesting and somewhat counterintuitive that the bare scalar soft mass can be thought of as given by the wavefunction evaluated at the scale $\mu = \Lambda_R$ (appropriately continued into superspace).

A remark on anomaly-free generators is now in order. The wavefunctions $\mathcal{Z}_r$ in eq. (2.1) can be thought of as gauge fields for the maximal abelian subgroup $[\mathrm{U}(1)]^K$ of the full flavor group of the model. We can choose a basis of generators so that only one of the U(1)'s has an anomaly and the rest are anomaly-free. For a soft mass proportional to an anomaly-free generator, the RG evolution of the soft masses is simply determined by charge conservation. This implies that the mapping between the UV and IR soft masses is obtained simply by matching quantum numbers of the composite. For example, in SUSY QCD a soft term associated with baryon number has the UV form $m_{\tilde Q}^2 = -m_{\tilde{\bar Q}}^2 = m_0^2$. In the s-confining case $N_F = N_c + 1$, the low-energy masses for baryons and mesons are simply $m_{\tilde B}^2 = -m_{\tilde{\bar B}}^2 = N_c m_0^2$ and $m_{\tilde M}^2 = 0$.

We now consider briefly the extension of these results to theories with superpotentials in the UV theory. In this case, the anomalous U(1) symmetries considered above do not suffice to determine the exact dependence on the soft masses because there are additional invariants that can be formed using the superpotential couplings. For example, suppose that the UV theory contains a Yukawa coupling λ. In that case, we can define an additional RG invariant $\hat\lambda$ corresponding to the running Yukawa coupling renormalized at the scale Λ_R. The quantity $|\hat\lambda|^2$ is neutral under all symmetries (including $\mathrm{U}(1)_R$), and therefore symmetries do not suffice to determine how this quantity appears in the effective Kähler potential. We can of course use holomorphy and symmtries to determine the exact dependence of the effective superpotential on the Yukawa coupling. This can give nontrivial information in the case where the running Yukawa coupling is perturbative both at the scale Λ_R and at a UV scale μ_0 where the gauge coupling is also perturbative. (We cannot in general take $\mu_0 \to \infty$ because theories with Yukawa couplings are strongly coupled in the ultraviolet.) In that case, we can expand the RG invariants in powers of $|\lambda(\mu_0)|^2/(16\pi^2)$, and 'naïve dimensional analysis' [27, 28] tells us that the effective Kähler potential is an expansion in $|\hat{\lambda}|^2/(16\pi^2)$.[2] If $\lambda(\mu_0)$, $\hat{\lambda} \ll 4\pi$, these effects are smaller than the 'tree-level' dependence on the Yukawa coupling in the effective superpotential.

Similar remarks apply to the case where the theory has a product gauge group, with some matter fields charged under multiple group factors. If one of the gauge couplings becomes strong at a scale where all the other gauge couplings are weak, we can compute the effects of soft masses up to perturbative corrections using ideas similar to those discussed above for Yukawa couplings. We cannot treat the case where several factors of the gauge group become strong at the same scale.[3]

There are special choices of soft masses for which the RG-invariant Yukawa couplings $\hat{\lambda}_i$ or ratios of strong scales $\Lambda_{Ra}/\Lambda_{Rb}$ have no θ dependence. In this case the flow of soft terms can be controlled as in our simple SQCD examples. The physical interpretation of these special RG trajectories is is clarified below using a gauged $\mathrm{U}(1)_R$ background.

2.2 Deformed moduli space

In ref. [8], this formalism was applied to theories with confining and infrared free 'dual' descriptions. We now apply these results to soft breaking in theories with deformed moduli spaces. We begin with SU(2) SUSY QCD with 4 fundamentals Q^j, $j = 1, \ldots, 4$ (2 'flavors'). In the SUSY limit, the moduli space can be parameterized by the holomorphic gauge-invariants ('mesons')

$$M^{jk} = \frac{1}{\Lambda_S^2} Q^j Q^k = -M^{kj} \,. \tag{2.28}$$

Classically, these satisfy the constraint $\mathrm{Pf}(M) = 0$, but this is modified by quantum effects to [2] (see also ref. [17])

$$\mathrm{Pf}(M) = 1 \,. \tag{2.29}$$

The anomaly-free $\mathrm{U}(1)_R$ charge of Q vanishes and the anomalous U(1) charge of Q and Λ are the same, so the quantum constraint is consistent with all symmetries.

[2]Since $\hat{\lambda}$ differs from $\lambda(\mu_0)$ by an RG factor of order $\lambda^3(\mu_0)\ln(\mu_0/\Lambda_R)/(16\pi^2)$, there are large logarithms in the expansion when expressed in terms of $\lambda(\mu_0)$.

[3]It may be possible to make progress in theories with discrete symmetries that interchange the gauge group factors.

To simplify the analysis, we use the (Lie algebra) isomorphism between SU(4) and SO(6). In SO(6) language, we write the mesons as M_a, $a = 1, \ldots, 6$ with constraint

$$M_a M_a = 1\,. \tag{2.30}$$

If the soft breaking masses are small compared to the dynamical scale Λ of the theory, they will make a small perturbation on the SUSY moduli space. We therefore write the most general SO(6) invariant effective lagrangean written in terms of fields M satisfying the constraint eq. (2.30). The only SO(6) invariant combinations of M are $M_a^\dagger M_a$ and $M_a M_a = 1$ (by the quantum constraint), so we have

$$\mathcal{L}_{\text{eff}} = \int d^2\theta d^2\bar{\theta}\, \Lambda_R^2\, k(M^\dagger M) + \text{derivative terms}\,, \tag{2.31}$$

where M satisfies eq. (2.30). Note that $M^\dagger M$ is completely neutral: it is invariant under SO(6), $\mathrm{U}(1)_R$, and the anomalous U(1), and is also dimensionless. To calculate with this effective lagrangean, we must choose independent fields to parameterize M so that the constraint eq. (2.30) is satisfied. Expanding in these fields gives the terms in the effective lagrangean in terms of derivatives of the function k. The function k is completely unknown, except that it must give positive definite kinetic terms when expanded about any point.

Up to SO(6) rotations the most general VEV can be written as

$$\langle M \rangle = \begin{pmatrix} (1+v^2)^{1/2} \\ iv \\ 0 \\ \vdots \\ 0 \end{pmatrix}, \tag{2.32}$$

where $-\infty < v < +\infty$ parameterizes the set of inequivalent vacua in the SUSY limit. For $v \neq 0$, SO(6) is broken to SO(4), while at $v = 0$ the unbroken symmetry is enhanced to SO(5). We then write

$$M = \langle M \rangle (1 + \Delta) + \Phi\,, \qquad \langle M_a \rangle \Phi_a = 0\,. \tag{2.33}$$

The constraint eq. (2.30) can be solved to give

$$\Delta = -\frac{1}{2}\Phi^2 + O(\Phi^4)\,. \tag{2.34}$$

Using the basis

$$\Phi = \frac{1}{(1+2v^2)^{1/2}} \begin{pmatrix} v \\ i(1+v^2)^{1/2} \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \Phi_1 + \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \Phi_2 + \cdots + \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \Phi_5 \tag{2.35}$$

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we have

$$M^\dagger M = \langle M^\dagger M\rangle + \frac{2v(1+v^2)^{1/2}}{(1+2v^2)^{1/2}}(\Phi_1 + \Phi_1^\dagger) - \frac{1}{2}(\Phi - \Phi^\dagger)^2 + O(\Phi^3)\,. \tag{2.36}$$

Note that $\langle M^\dagger M\rangle = 1 + 2v^2$.

The vacuum energy as a function of v can be determined from the terms in the effective potential that are independent of the scalar components of Φ. Eliminating the auxiliary components of Φ, we obtain

$$V(v) = -[\Lambda_R^2]_{\theta^2\bar\theta^2}\langle k\rangle + \frac{|[\Lambda_R^2]_{\theta^2}|^2}{[\Lambda_R^2]_0}\frac{4v^2(1+v^2)\langle k'\rangle^2}{(1+2v^2)\langle k'\rangle + 4v^2(1+v^2)\langle k''\rangle}\,, \tag{2.37}$$

where $\langle k\rangle = k(1+2v^2)$, etc. We do not know the function k explicitly, but we know that k' must be nonzero everywhere on the moduli space in order for the kinetic terms to be positive in the SUSY limit. This is sufficient to conclude that the enhanced symmetry point $v = 0$ is a local minimum for any positive soft scalar mass ($[\Lambda_R^2]_{\theta^2\bar\theta^2} < 0$) and for any gaugino mass ($[\Lambda_R]_{\theta^2}^2$).

In the absence of a gaugino mass only the first term in eq. (2.37) survives, so that the positivity of k' is sufficient to conclude that the global minimum is at $v = 0$. In this case, the effective lagrangean is

$$\mathcal{L}_{\text{eff}} = \int d^2\theta d^2\bar\theta\,\Lambda_R^2\,k'(1)\left[\Phi_a^\dagger\Phi_a - \frac{1}{2}(\Phi_a\Phi_a + \text{h.c.})\right] + O(\Phi^4)\,. \tag{2.38}$$

Note that at this order the only dependence on the effective Kähler potential is through an overall factor, which cancels when we compute masses. The masses are

$$m^2_{\text{Re}(\Phi)} = 0\,, \qquad m^2_{\text{Im}(\Phi)} = +2m_0^2\,, \tag{2.39}$$

where m_0^2 is a common bare soft mass term of the fundamental Q fields. The fields $\text{Re}(\Phi)$ are massless because they are the Nambu-Goldstone bosons of the global symmetry breaking $\text{SO}(6) \to \text{SO}(5)$. (Local $\text{SO}(6)$ excitations of the VEV eq. (2.32) correspond to the real components of Φ.) The mass of the $\text{Im}(\Phi)$ fields is a simple multiple of the bare mass, and is positive if the bare mass is positive.

These techniques can be applied to other models with deformed moduli spaces, but we cannot generally determine the scalar masses. Consider for example the case of $\text{SU}(N)$ gauge theory ($N \geq 3$) with N 'flavors' of quarks $Q^j, \bar Q_{\bar k}$, $j, \bar k = 1, \ldots, N$. In the SUSY limit, the moduli space is parameterized by the gauge invariants

$$M^j{}_{\bar k} \equiv \frac{1}{\Lambda_S^2}Q^j\bar Q_{\bar k}\,, \qquad B \equiv \frac{1}{\Lambda_S^N}\det Q\,, \qquad \bar B \equiv \frac{1}{\Lambda_S^N}\det\bar Q\,. \tag{2.40}$$

The quantum constraint is

$$\det M - B\bar B = 1\,. \tag{2.41}$$

The most general effective lagrangean invariant under the $\mathrm{U}(N) \times \mathrm{U}(N)$ flavor symmetry (which includes the anomalous $\mathrm{U}(1)$) and the anomaly-free $\mathrm{U}(1)_R$ symmetry is

$$\mathcal{L}_{\text{eff}} = \int d^2\theta d^2\bar{\theta}\, \Lambda_R^2\, k\left(B^\dagger, B, \bar{B}^\dagger, \bar{B}, M, M^\dagger\right) + \text{derivative terms}\,, \tag{2.42}$$

where M, B, and $\bar{B}$ satisfy the quantum constraint eq. (2.41). (The content of the above equation is that Λ_R appears only as a multiplicative factor.)

Suppose we are interested in the maximally symmetric point $\langle B \rangle = \langle \bar{B} \rangle = 0$, $\langle M \rangle = 1$. We write

$$M = \langle M \rangle + \Phi\,, \tag{2.43}$$

and the constraint tells us that

$$\mathrm{tr}(\Phi) = B\bar{B} - \frac{1}{2}\,\mathrm{tr}(\Phi'^2) + \text{quadratic terms}\,, \tag{2.44}$$

where $\Phi' = \Phi - (\mathrm{tr}\,\Phi)/N$ is the trace-free part of Φ. No symmetry can forbid a term in the effective Kähler potential of the form

$$k \sim B\bar{B} + \text{h.c.} \tag{2.45}$$

(A quick way to see this is that the combination $B\bar{B}$ appears in the quantum constraint.) The quantum constraint is solved by eq. (2.44), and so this term is quadratic in terms of the independent fields. Therefore, in the presence of soft masses, a term of the form eq. (2.45) this term gives a 'B type' mass for the baryon fields, and we cannot determine the masses for these fields.

Although it is of limited interest, we can determine the mass-squared for the mesons in the maximally symmetric vacuum. The unbroken $\mathrm{U}(N)$ 'diagonal' symmetry means that the Kähler potential has the form

$$k = k_0 \left\{ \mathrm{tr}(\Phi'^\dagger \Phi') + c\left[\mathrm{tr}(\Phi'^2) + \text{h.c.}\right] + \cdots \right\}. \tag{2.46}$$

The fact that there must be $N^2 - 1$ Nambu-Goldstone bosons from the spontaneous symmetry breaking of the global symmetry means that $c = -1/2$, and the soft masses for the meson fields are determined. As in the $\mathrm{SU}(2)$ case, we find that the mass-squared for the $N^2 - 1$ massive mesons is $+\frac{1}{2}m_{\phi 0}^2$.

3. Anomaly-free gauged $\mathrm{U}(1)_R$

In this section we give a new derivation of the results of ref. [8] reviewed in the previous section that makes use of *anomaly-free* gauge symmetries. This gives additional insight into why we are able to obtain exact results for non-holomorphic quantities. We use these results to obtain a very simple derivation of the behavior of soft masses in a theory with a conformal fixed point.

The ideas are easiest to explain in the context of a $\mathcal{N}=1$ supergravity (SUGRA) background with a gauged $\mathrm{U}(1)_R$. This can be formulated simply using the superconformal approach to SUGRA [19]. In the flat limit, the tree-level lagrangean for a gauge theory coupled to a SUGRA background can be written in superspace as [20]

$$\mathcal{L} = \int d^4\theta \, (\phi^\dagger e^{-\frac{2}{3}V_R}\phi)(Q^\dagger \mathcal{Z} e^V e^{V_R \cdot R} Q) + \left(\int d^2\theta \, \phi^3 W(Q) + \text{h.c.} \right) + \\ + \int d^2\theta \, S \, \mathrm{tr}(W^\alpha W_\alpha) + \text{h.c.} \tag{3.1}$$

The chiral field ϕ is the conformal compensator, whose role in the full formalism is to break the superconformal symmetry down to super-Poincaré symmetry. The ϕ dependence is completely determined by dilatation invariance and $\mathrm{U}(1)_R$ invariance, under which ϕ has respectively weight $+1$ and charge $2/3$. All other fields have vanishing weight and $\mathrm{U}(1)_R$ charge.[4] In this approach to SUGRA, the $\mathrm{U}(1)_R$ symmetry is part of the superconformal group, and is gauged (the gauge field is the SUGRA vector auxiliary field). The field V_R is a gauge superfield for an *ordinary* $\mathrm{U}(1)$ gauge group whose charge matrix we call R. The superconformal compensator is charged under this $\mathrm{U}(1)$ with $R_\phi = -2/3$. The VEV of the conformal compensator $\phi = 1 + \cdots$ therefore breaks $\mathrm{U}(1) \times \mathrm{U}(1)_R$ down to the diagonal $\mathrm{U}(1)$ subgroup. This unbroken group is an R symmetry, and the matter fields have charge R. (This is the justification for the somewhat abusive notation used above, where the charge of the ordinary $\mathrm{U}(1)$ is denoted by R.) It is this $\mathrm{U}(1)$ symmetry that must be anomaly-free in order for the dependence on V_R to be fixed simply by considerations of charge conservation. As shown in ref. [18], the condition that a $\mathrm{U}(1)_R$ symmetry with charges R must be anomaly-free in order to define a consistent deformation of the theory can also be derived without referring to SUGRA.

We now consider the SUSY-breaking background

$$V_R = \theta^2 \bar\theta^2 D_R \,, \qquad \phi = 1 + \theta^2 F_\phi \,. \tag{3.2}$$

The field D_R gives rise to soft masses at tree level, but the dependence on F_ϕ is more subtle. Note that if the lagrangean contains no dimensionful terms, then $W(Q) \sim Q^3$ and the ϕ dependence can be completely eliminated from the tree-level lagrangean by a field redefinition $Q' = \phi Q$. However, regulating the theory necessarily introduces mass parameters and therefore brings in additional ϕ dependence at loop level [21, 22]. The coupling of ϕ is completely determined by dilatation symmetry, so the loop effects are correctly included by the replacement

$$\mu^2 \to \hat\mu^2 = \frac{\mu^2}{\phi^\dagger e^{-\frac{2}{3}V_R}\phi} \tag{3.3}$$

[4] In fact, it is worth noting that we do not need SUGRA to determine the dependence of ϕ. However, SUGRA makes the dependence of V_R more clear.

in the renormalized couplings $R(\mu) = 1/g^2(\mu)$ and $\mathcal{Z}_r(\mu)$. This gives rise to running scalar and gaugino masses [18]

$$m_r^2(\mu) = -[\ln \mathcal{Z}_r(\hat{\mu})]_{\theta^2\bar{\theta}^2} = \left(\frac{2}{3} - R_r - \frac{1}{3}\gamma_r\right) D_R - \frac{1}{4}\frac{d\gamma_r}{d\ln\mu}|F_\phi|^2\,, \tag{3.4}$$

$$m_\lambda(\mu) = [\ln R(\hat{\mu})]_{\theta^2} = \frac{\beta(g^2)}{2g^2}F_\phi\,, \tag{3.5}$$

where $\gamma_r = d\ln Z_r/d\ln\mu$ and $\beta(g^2) = dg^2/d\ln\mu$. These equations define a consistent RG trajectory to all orders in perturbation theory in an appropriate class of renormalization schemes [13]. The 'bare' soft mass parameters on this RG trajectory are

$$\begin{aligned} m_{r0}^2 &= \lim_{\mu\to\infty} m_r^2(\mu) = \left(\frac{2}{3} - R_r\right) D_R\,, \\ \frac{m_{\lambda 0}}{g_0^2} &= \lim_{\mu\to\infty}\frac{m_\lambda(\mu)}{g^2(\mu)} = -\frac{b}{16\pi^2}F_\phi\,, \end{aligned} \tag{3.6}$$

where $b = 3t_G - \sum_r t_r$ is the coefficient of the gauge beta function.

For $\mathrm{SU}(N_c)$ SUSY QCD with N_f flavors,

$$m_{r0}^2 = \frac{3N_c - N_f}{3N_f}D_R\,, \tag{3.7}$$

$$\frac{m_{\lambda 0}}{g_0^2} = -\frac{3N_c - N_f}{16\pi^2}F_\phi\,, \tag{3.8}$$

We can now apply these results to the low-energy effective theory to find the mapping of the UV soft masses onto IR soft masses. For $N_c + 1 < N_f < \frac{3}{2}N_c$ the low-energy description has an infrared-free 'dual' description in terms of an $\mathrm{SU}(N_f - N_c)$ gauge theory with dual quarks $q, \bar{q}$ and neutral 'meson' fields M. Because this theory is infrared-free, we can easily read off the soft masses of these fields on the RG trajectory defined above in the far infrared:

$$\begin{aligned} m_{q,\mathrm{IR}}^2 &= \lim_{\mu\to 0} m_q^2(\mu) = \left(\frac{2}{3} - R_q\right) D_R = \frac{2N_f - 3N_c}{3N_f}D_R\,, \\ m_{M,\mathrm{IR}}^2 &= \lim_{\mu\to 0} m_M^2(\mu) = 2\frac{3N_c - 2N_f}{3N_f}D_R\,, \\ \left.\frac{m_{\lambda_D}}{g_D^2}\right|_{\mathrm{IR}} &= \lim_{\mu\to 0}\frac{m_{\lambda_D}(\mu)}{g_D^2(\mu)} = -\frac{2N_f - 3N_c}{16\pi^2}F_\phi\,, \end{aligned} \tag{3.9}$$

where λ_D is the dual gaugino and g_D the dual gauge coupling. Comparing to eqs. (3.7) and (3.8), we obtain the relation between the UV and IR soft masses obtained in ref. [8]. (The equations above are valid also in the s-confining case $N_f = N_c + 1$, where the dual quarks are identified with the baryons.) Of course, the physical masses should be evaluated at a renormalization scale μ equal to the physical mass. However, this will give corrections to the masses of order $g_D^2(\mu)/16\pi^2$, where $g_D(\mu)$ is the running coupling in the dual description. These corrections are small if the dual description is weak.

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The gauged non-anomalous $U(1)_R$ is also interesting for theories with Yukawa couplings or multiple gauge factors. Here it can be used to define some non-trivial, but nonetheless 'integrable', soft term RG flow. Indeed the duals of pure gauge theories often involve Yukawa couplings. The underlying $U(1)_R$ symmetry then makes it more clear why in ref. [8] the soft term flow of the dual theory could also be followed exactly.

We now consider SUSY QCD in the conformal window $\frac{3}{2}N_c \leq N_f \leq 3N_c$. This was considered in ref. [9], where the explicit RG equations of ref. [23] were used to show that all soft masses scale to zero. In this approach, the origin of this result is clouded in the computations; we believe that the supergravity approach gives a significant clarification.

First, it is obvious that when the theory approaches a scale invariant point the dependence on the scale compensator ϕ must drop out from the effective action. This is manifest in eq. (3.5), since the contribution of F_ϕ is proportional to $\beta(g^2)$, which vanishes at the fixed point. Second, if we choose R to be the same for all quark fields, the contribution of D_R is proportional to

$$2 - 3R_Q - \gamma_Q \propto 3N_c - N_f - N_f\gamma_Q\,, \tag{3.10}$$

which is the quantity that controls the vanishing of the NSVZ beta function:

$$\mu\frac{dg^2}{d\mu} = -\frac{g^4}{8\pi^2}\frac{b - \sum\limits_r t_r\gamma_r}{1 - \dfrac{g^2}{8\pi^2}t_G}\,. \tag{3.11}$$

This result holds because the R charge chosen is identical with the $U(1)_R$ charge in the superconformal algebra at the fixed point, which satisfies $d_{\mathcal{O}} = \frac{3}{2}R_{\mathcal{O}}$ for any chiral operator $\mathcal{O}$ with scaling dimension $d_{\mathcal{O}}$. For example, the $\mathcal{O}$ 2-point function can be described by a term in the 1PI effective action

$$\Gamma_{\text{1PI}} \propto \int d^4\theta\ \left(\mathcal{O}^\dagger e^{V_R R_{\mathcal{O}}}\Box^{1-d_{\mathcal{O}}}\mathcal{O}\right)\left(\phi^\dagger e^{-\frac{2}{3}V_R}\phi\right)^{d_{\mathcal{O}}} + \cdots \tag{3.12}$$

in which the dependence on V_R drops out.

These results give the scaling of the soft masses for μ larger than the soft masses themselves; below this scale, the soft masses are relevant perturbations and the physics is no longer controlled by the fixed point. The approach to the fixed point $g = g_*$ is given by

$$g^2(\mu) = g_*^2 + c\left(\frac{\mu}{\Lambda}\right)^{\gamma'}\,, \tag{3.13}$$

where Λ is the scale of strong interactions. The critical exponent is

$$\gamma' = \left(1 - \frac{g_*^2}{8\pi^2}t_G\right)^{-1}\sum_r \frac{g_*^2}{8\pi^2}t_r\,\frac{d\gamma_r}{d\ln g^2}\bigg|_* > 0\,. \tag{3.14}$$

In a strongly-coupled theory, naïve dimensional analysis tells us that $\gamma' \sim 1$. By eqs. (3.4) and (3.5) we find that the scaling of soft terms is $m_Q(\mu) \sim F_\Phi(\mu/\Lambda)^{\gamma'/2}$ and $m_\lambda(\mu) \sim F_\Phi(\mu/\Lambda)^{\gamma'}$. We see that, for $\mu \ll \Lambda$, $m_Q \gg m_\lambda$, so the scalar masses control the exit from the fixed point. Solving $m_Q^2(\mu)(\mu \sim m_Q) \sim m_Q$ gives

$$\frac{m_Q}{\Lambda} \sim \left(\frac{F_\phi}{\Lambda}\right)^{2/(2-\gamma')}, \qquad \frac{m_\lambda}{\Lambda} \sim \left(\frac{F_\phi}{\Lambda}\right)^{(2+\gamma')/(2-\gamma')}, \tag{3.15}$$

where we have assumed that the gaugino masses essentially freeze upon exiting the fixed point. For $\gamma' > 2$, this solution is not applicable. In that case, the scalar mass is scaling to zero faster than μ itself, and the physical soft masses vanish. This is a logical possibility in strongly-coupled theories, but unfortunately we are unable to compute γ' and so we cannot determine whether this occurs.[5]

We close this section with some remarks on the possible phenomenological applications of conformal theories. The fact that soft masses decrease as a non-trivial power law in the infrared in stronlgly-coupled conformal theories raises the possibility that this could play a role in understanding the smallness of SUSY breaking in our world. However, there are some very generic difficulties with this idea. First, as pointed out above, the gaugino masses are always smaller than the scalar masses in such a scenario. Second, the reduction of the scalar mass discussed above applies only to the component proportional to the anomaly-free $\mathrm{U}(1)_R$ generator. All flavor breaking scalar masses associated to the anomaly-free generators of the flavor group ($\mathrm{SU}(N_f) \times \mathrm{SU}(N_f)$ for SQCD) will not undergo the suppression discussed above. Since realistic supersymmetric theories require the squark masses to be approximatively flavor-preserving, this will make the SUSY flavor problem more severe. However, strongly-coupled theories near their conformal fixed points may play a role in nature for other reasons, and it is important to know how the soft masses scale in such theories.

4. $\mathcal{N} = 2$ super-Yang-Mills

We now turn our attention to $\mathrm{SU}(2)$ $\mathcal{N} = 2$ super-Yang-Mills theory.

4.1 $\mathcal{N} = 2$ spurion analysis

We first consider the theory formulated in $\mathcal{N} = 2$ superspace and perform a spurion analysis by generalizing the couplings to $\mathcal{N} = 2$ superfields. This analysis generalizes the results of ref. [7] because we use more general $\mathcal{N} = 2$ spurions. This is sufficient

[5] The anomalous dimension γ' can be calculated in weakly coupled theories, such as the $1/N_c$ expansion around the Banks-Zaks fixed point at $3N_c - N_f = \epsilon N_c$ in SQCD. There one finds $\gamma' \sim \epsilon^2 \ll 1$ [9]. The question seems to be open whether $\gamma' > 2$ in the middle of the conformal window.

to parameterize all soft SUSY breaking except for a non-holomorphic soft mass for the scalar field. We work out the effects of this breaking on the low-energy potential using $\mathcal{N} = 2$ techniques and compare our results to those of ref. [7]. We also give an elementary derivation of the relation between the modulus and the prepotential.

The gauge multiplet can be described in $\mathcal{N} = 2$ superspace $(x^\mu, \theta^\alpha, \tilde{\theta}^\alpha, \theta^\dagger_{\dot\alpha}, \tilde{\theta}^\dagger_{\dot\alpha})$ by a superfield $\mathcal{A}$ satisfying [25]

$$\bar{D}^j_{\dot\alpha}\mathcal{A} = 0\,, \tag{4.1}$$

$$D^i D^j \mathcal{A} = \bar{D}^i \bar{D}^j \bar{\mathcal{A}}\,, \tag{4.2}$$

where i, j are $\mathrm{SU}(2)_R$ indices ($\theta^\alpha_1 = \theta^\alpha$, $\theta^\alpha_2 = \tilde{\theta}^\alpha$).[6] Notice that by $\mathrm{SU}(2)_R$ covariance $\bar{D}^i = \epsilon^{ij}(D^j)^*$, where ϵ^{ij} is the antisymmetric tensor. eq. (4.1) states that $\mathcal{A}$ is a $\mathcal{N} = 2$ chiral multiplet, while eq. (4.2) is a reality condition that defines an $\mathcal{N} = 2$ vector multiplet. In this notation, the lagrangean is

$$\mathcal{L} = \int d^2\theta d^2\tilde{\theta} \frac{1}{2g^2}\,\mathrm{tr}(\mathcal{A}^2) + \mathrm{h.c.} \tag{4.3}$$

(We have set the theta term to zero. We take $\mathcal{A} = \mathcal{A}_b \tau_b$ with the SU(2) generators normalized by $\mathrm{tr}(\tau_a\tau_b) = \frac{1}{2}\delta_{ab}$.) The $\mathcal{N} = 1$ decompositions of $\mathcal{A}$ is

$$\mathcal{A} = \Phi + i\sqrt{2}\tilde{\theta}^\alpha W_\alpha + \tilde{\theta}^2 \left[-\frac{1}{4}\bar{D}^2 \left(e^V \Phi e^{-V} \right) \right], \tag{4.4}$$

where Φ and W_α are $\mathcal{N} = 1$ chiral superfields that are functions of θ and

$$y^\mu = x^\mu + i\left(\theta\sigma^\mu\theta^\dagger + \tilde{\theta}\sigma^\mu\tilde{\theta}^\dagger \right). \tag{4.5}$$

We now consider extending the gauge coupling to a $\mathcal{N} = 2$ superfield:

$$\mathcal{L} = \int d^2\theta d^2\tilde{\theta}\, \Sigma\, \mathrm{tr}(\mathcal{A}^2) + \mathrm{h.c.} \tag{4.6}$$

This is $\mathcal{N} = 2$ supersymmetric provided that Σ is chiral:

$$\bar{D}^j_{\dot\alpha}\Sigma = 0\,. \tag{4.7}$$

Ref. [7] also performed a $\mathcal{N} = 2$ spurion analysis, but they imposed the additional condition that Σ is a $\mathcal{N} = 2$ vector multiplet.

The low-energy effective theory arising from the strong dynamics depends on Σ through the RG-invariant scale

$$\Lambda = \mu e^{-4\pi^2\Sigma(\mu)}\,, \tag{4.8}$$

[6] We use the conventions of ref. [25], which extend the conventions of Wess and Bagger [26] to $\mathcal{N} = 2$ superspace. For a precise definition of $\bar{\mathcal{A}}$, see ref. [25].

where we have used the beta function appropriate for SU(2). (Recall that $\mathcal{N} = 2$ SUSY implies that the gauge coupling runs only at one loop.) Note that this is an $\mathcal{N} = 2$ chiral superfield. Away from the monopole points, the effective theory can be written in terms of a U(1) gauge superfield a:

$$\mathcal{L}_{\text{eff}} = \frac{1}{4\pi} \operatorname{Im} \int d^2\theta d^2\tilde{\theta}\, \mathcal{F}(a, \Lambda)\,, \tag{4.9}$$

where dimensional analysis implies

$$\mathcal{F}(a, \Lambda) = \Lambda^2 \mathcal{G}(a/\Lambda)\,. \tag{4.10}$$

These considerations lead directly to an elementary proof of the relation between the modulus $u \equiv \operatorname{tr} \phi^2$ and the prepotential. Suppose we turn on a $\theta^2\tilde{\theta}^2$ component of Σ as a source:

$$\Sigma = \frac{1}{2g^2} + \theta^2\tilde{\theta}^2\tilde{D}\,. \tag{4.11}$$

From the fundamental lagrangean, we see that

$$\left.\frac{\delta\Gamma_{\text{1PI}}}{\delta\tilde{D}}\right|_{\tilde{D}=0} = \langle u \rangle\,. \tag{4.12}$$

In the effective theory, we can evaluate the term linear in $\tilde{D}$ in $\delta\Gamma_{\text{1PI}}/\delta\tilde{D}$ by expanding out the θ- and $\tilde{\theta}$-dependent terms in Λ:

$$\begin{aligned}
\tilde{D}\langle u \rangle &= \frac{1}{8\pi i}\left[\Lambda^2\mathcal{G}(a/\Lambda)\right]_{\theta^2\tilde{\theta}^2} \\
&= \frac{1}{8\pi i}\left[[\Lambda^2]_{\theta^2\tilde{\theta}^2}\mathcal{G}(a/\Lambda) + \Lambda^2\mathcal{G}'(a/\Lambda)\cdot a[1/\Lambda]_{\theta^2\tilde{\theta}^2}\right] \\
&= \frac{\pi}{2i}\left[\tilde{D}\,(2\mathcal{F} - aa_D)\right],
\end{aligned} \tag{4.13}$$

where $a_D \equiv \partial\mathcal{F}/\partial a$. This immediately gives

$$\mathcal{F} - \frac{1}{2}aa_D = -\frac{i}{\pi}u\,. \tag{4.14}$$

Previous derivations [10] of this result have relied on specific properties of the Seiberg-Witten solution. Here we see that it follows from elementary spurion considerations.

We can also use this formalism to work out the effects of soft SUSY breaking in the low-energy theory. If we write

$$\Sigma = \frac{1}{g^2}\left[\frac{1}{2} + \theta^2 m_\lambda + \tilde{\theta}^2 m_\chi + 2i\theta\tilde{\theta}m_D - \theta^2\tilde{\theta}^2 m_B^2\right] \tag{4.15}$$

then this gives rise to SUSY breaking terms in the fundamental lagrangean:

$$\Delta\mathcal{L} = \frac{1}{g^2}\operatorname{tr}\left[-m_\lambda\lambda\lambda - m_\chi\chi\chi - 2im_D\lambda\chi - (2g^4\Delta + m_B^2)\phi^2\right] + \text{h.c.} - 2g^2 T\operatorname{tr}(\phi\phi^*)\,, \tag{4.16}$$

where

$$T = \frac{1}{g^4}\,\mathrm{tr}(m_\Psi^\dagger m_\Psi)\,, \qquad \Delta = \frac{1}{g^4}\,\det(m_\Psi)\,, \qquad m_\Psi = \begin{pmatrix} m_\chi & -im_D \\ -im_D & m_\lambda \end{pmatrix}. \tag{4.17}$$

and where λ and χ are the fermion components of $\mathcal{A}$. (Because the kinetic terms are multiplied by a factor of $1/g^2$, the mass parameters as defined here are the running masses. In $\mathcal{N}=2$ there is no running beyond 1-loop, or, equivalently, the holomorphic and 1PI coupling can be take to coincide, $R = S + S^\dagger$. As a reflection of that the matrix m_Ψ/g^2 is RG invariant, and coincides with the bare parameter at $\mu = \infty$ defined in section 2.) We can now work out the effects of this soft SUSY breaking in the low-energy theory directly from the Seiberg-Witten solution for $\mathcal{G}$ in (4.10) by expanding out the θ- and $\tilde\theta$-dependent terms in Λ.

Specifically, the terms relevant for the potential for the effective theory are

$$a = \sigma + \theta^2 f + \tilde\theta^2 f^\dagger + i\theta\tilde\theta d\,, \tag{4.18}$$

where σ is the complex propagating scalar, and f, d are auxiliary fields. The reality condition on a implies that d is real, relates the coefficients of the θ^2 and $\tilde\theta^2$ terms, and implies the absence of a $\theta^2\tilde\theta^2$ component. The effective lagrangean including soft SUSY breaking is then simply given by eq. (4.9), where we expand the θ- and $\tilde\theta$ dependence of both a and Λ. After some straighforward algebra (and use of eq. (4.14)), we obtain

$$V_{\mathrm{eff}} = \frac{T}{k_{u^\dagger u}} + 8\pi^2\,\mathrm{Re}\left[\Delta\cdot\left(2u - \frac{k_{u^\dagger}}{k_{u^\dagger u}}\right)\right] + \frac{2}{g^2}\,\mathrm{Re}(m_B^2 u)\,, \tag{4.19}$$

Note that $\mathrm{SU}(2)_R$ is manifest. The results above are valid away from the monopole points; we will postpone the discussion of the physics of this result to the next section, where we consider the most general soft breaking terms.

Before we leave the subject of $\mathcal{N}=2$ spurions, we comment on the relation between our results and those of ref. [7]. In that paper the 'dilaton' $S = i\Sigma$ is taken to be a vector superfield. This corresponds to setting $m_B^2 = 0$, and $m_\lambda = -m_\psi^\dagger$ with m_D pure imaginary, which gives $T = -2\Delta$. $\mathrm{SU}(2)_R$ invariance means that the perturbation is to just one independent soft parameter T. From eq. (4.16) it is manifest that for this choice of parameters the half-line $\mathrm{tr}\,\phi^2 = \mathrm{tr}(\phi\phi^\dagger) > 0$ remains flat to all orders in perturbation theory. Non-perturbative effects remove this flatness and the vacuum is picked out along this half-line at the monopole point $u = 1$ [7]. Notice that if we had taken Σ rather that $i\Sigma$ to be vectorlike, the flat direction would have been along $\mathrm{tr}\,\phi^2 = -\,\mathrm{tr}(\phi\phi^\dagger) < 0$, and the vacuum would be stabilized at the dyon point $u = -1$. This explains the apparent asymmetry between the monopole and dyon points in ref. [7].[7]

[7]In fact, under the Z_8 R-symmetry of the theory, we have $u \mapsto -u$, $\Sigma(\theta,\tilde\theta) \mapsto \Sigma(\theta e^{i\pi/4}, \tilde\theta e^{i\pi/4})$. Therefore the symmetry that exchanges monopole and dyon poins is $m_\Psi \to i m_\Psi$, $(T,\Delta) \mapsto (T,-\Delta)$.

4.2 General spurion analysis

We now consider the most general soft breaking down to $\mathcal{N}=0$. In $\mathcal{N}=1$ superspace, the action including the most general soft breaking terms can be written

$$\begin{aligned}\mathcal{L} = &\int d^2\theta d^2\bar{\theta}\, \mathcal{Z}_\Phi \operatorname{tr}(\Phi^\dagger e^V \Phi e^{-V}) + \left(\int d^2\theta\, S \operatorname{tr}(W^\alpha W_\alpha) + \text{h.c.}\right) + \\ &+ \left(\int d^2\theta\, m \operatorname{tr}(\Phi^2) + \text{h.c.}\right),\end{aligned} \tag{4.20}$$

where $\mathcal{Z}_\Phi$, S, and m are now regarded as superfields with θ-dependent components parameterizing the SUSY breaking. Because Φ transforms as an adjoint under the gauge group, there is an additional allowed soft term of the form

$$\int d^2\theta\, \theta^\alpha \operatorname{tr}(W_\alpha \Phi) + \text{h.c.}\,, \tag{4.21}$$

which gives rise to a mixing mass between the gaugino and Φ fermion.[8] However, the theory has a $\mathrm{SU}(2)_R$ symmetry that is not manifest in the $\mathcal{N}=1$ formulation under which the gaugino and Φ fermion form a doublet. We therefore choose the '$\mathcal{N}=1$' direction to diagonalize the fermion mass matrix and eliminate the term eq. (4.21). Under $\mathrm{SU}(2)_R$, the fermions χ and λ form a doublet

$$\Psi^j = \begin{pmatrix} \chi \\ \lambda \end{pmatrix}, \tag{4.22}$$

and the fermion masses form a triplet

$$(m_{\Psi 0})_{jk} = \begin{pmatrix} m_{\chi 0} & 0 \\ 0 & m_{\lambda 0} \end{pmatrix}. \tag{4.23}$$

The $\mathrm{SU}(2)_R$ invariance of our results will be a non-trival check of our formalism.

In the $\mathcal{N}=2$ SUSY limit, we have $\mathcal{Z}_\Phi \to S + S^\dagger$, and therefore $R \to S + S^\dagger$, $\Lambda_R^2 \to |\Lambda_S|^2$. We now add the most general soft SUSY breaking terms: a scalar mass m_ϕ^2 for the scalar component of Φ; a B-type mass term $B \operatorname{tr}\phi^2 + \text{h.c.}$; and fermion masses m_λ and m_χ, where λ is the gaugino and χ is the fermion component of Φ. As discussed above, these terms can be viewed as θ-dependent terms in the superfield coupling constants. As might be expected, this technique is especially powerful in theories with $\mathcal{N}=2$ SUSY. For example, using the gauged $\mathrm{U}(1)_R$ or the results of section 2, one finds

$$\mathcal{Z}_\Phi(\mu) = \hat{\mathcal{Z}}_\Phi R(\mu) = \hat{\mathcal{Z}}_\Phi \left(S(\mu) + S^\dagger(\mu) - \frac{1}{4\pi^2}\ln \hat{\mathcal{Z}}_\Phi\right), \tag{4.24}$$

[8] This term also gives a term in the scalar potential proportional to $\operatorname{tr}(\phi[\phi^\dagger,\phi])$+h.c. that vanishes identically.

where $\hat{\mathcal{Z}}_\Phi$ is the RG invariant wave function of eq. (2.13). In the $\mathrm{U}(1)_R$ approach one has $e^{-2V_R/3} = \hat{\mathcal{Z}}_\Phi$. This gives a simple exact closed-form expression for the running soft parameters.

The bare parameters $m^2_{\phi 0}$ and $m_{\lambda 0}$ are defined as in section 2. Moreover we define in the same spirit

$$m = \frac{m_{\chi 0}}{g_0^2} - \theta^2 \frac{m^2_{B0}}{g_0^2} \tag{4.25}$$

in a field basis where $[\ln \mathcal{Z}_\Phi]_{\theta^2} = 0$.

As long as the soft SUSY breaking parameters are small in units of Λ, they can be treated as a perturbation on the strong dynamics. These lift the flat directions and give a potential on the moduli space of SUSY vacua that we will determine. The moduli space can be parameterized by the chiral gauge-invariant operator

$$u \equiv \frac{1}{\Lambda_S^2} \operatorname{tr}(\Phi^2)\,. \tag{4.26}$$

Note that Λ_S and Φ transform in the same way under the anomalous U(1), so u is completely neutral: it is dimensionless, and uncharged under the anomaly-free $\mathrm{U}(1)_R$ symmetry

$$\Phi(\theta) \mapsto \Phi(\theta e^{-i\alpha})\,, \qquad V(\theta) \mapsto V(\theta e^{-i\alpha})\,, \qquad m(\theta) \mapsto e^{2i\alpha} m(\theta e^{-i\alpha})\,, \tag{4.27}$$

as well as the anomalous U(1) symmetry

$$\Phi \mapsto e^A \Phi\,, \qquad V \mapsto V\,, \qquad \Lambda_S \mapsto e^A \Lambda_S\,, \qquad m \mapsto e^{-2A} m\,. \tag{4.28}$$

Neutral variables similar to u are also present in the $\mathcal{N}=1$ theories with deformed moduli spaces discussed in section 2.2. This is no accident, since the moduli space is in a sense 'deformed' in the Seiberg-Witten solution, allowing the holomorphic prepotential to be a nontrivial meromorphic function of u.

4.3 Away from the monopole points

We begin by describing the theory away from the monopole/dyon points $\langle u \rangle = \pm 1$. As long as $|\langle u \rangle - (\pm 1)| \gtrsim 1$, the only light (compared to Λ) states in the theory are the U(1) gauge multiplet and the modulus field u. The most general effective lagrangean compatible with $\mathcal{N}=1$ supersymmetry, holomorphy, and anomalous U(1) invariance is

$$\begin{aligned}\mathcal{L}_{\text{eff}} = &\int d^2\theta d^2\bar\theta\, \Lambda_R^2 k(u^\dagger, u) + \left(\int d^2\theta\, \frac{1}{2} s(u) w^\alpha w_\alpha + \text{h.c.} \right) + \\ &+ \left(\int d^2\theta\, \Lambda_S^2 m u + \text{h.c.} \right) + \text{derivative terms}\,,\end{aligned} \tag{4.29}$$

where w_α is the U(1) gauge field strength. Note that all SUSY breaking is contained in the spurions Λ_R and Λ_S, as follows purely from $\mathcal{N}=1$ reasoning.

The Kähler function k cannot be determined from $\mathcal{N} = 1$ considerations, but it is completely fixed by the Seiberg-Witten solution in the $\mathcal{N} = 2$ SUSY limit. It is crucial for our results that the Seiberg-Witten solution also determines the purely chiral (or antichiral) part of the Kähler potential. (Recall that our inability to fix such terms was responsible for our inability to determine the vacuum in $\mathcal{N} = 1$ theories with deformed moduli space discussed previously.) These terms vanish in the SUSY limit, but they contribute to the potential when SUSY is broken explicitly. In the SUSY limit, these terms can be probed by promoting the $\mathcal{N} = 2$ gauge coupling superfield to a dilaton source, as done in ref. [7]. They are then fixed by the modular invariance of the Seiberg-Witten solution in the presence of the dilaton, which ensures that if we travel around a closed path in the moduli space we return to the same theory up to a duality transformation. Adding a chiral plus antichiral term to the effective superpotential corresponds to modifying the $\mathcal{N} = 2$ prepotential by $\mathcal{F}(a) \to \mathcal{F}(a) + \text{const} \times a$, but this clearly breaks modular invariance. (More direct physical arguments that the linear terms in the prepotential are fixed are also given in ref. [3].)

Combining this with the expressions for the higher components of Λ_R given in eqs. (2.23) and (2.25), we can compute the potential for the scalar component of u. The result is

$$V = |\Lambda|^2 \left(m_{\phi 0}^2 - \left| \frac{4\pi^2 m_{\lambda 0}}{g_0^2} \right|^2 \right) k - \left[\Lambda^2 \left(\frac{m_{B0}^2}{g_0^2} - 2 \frac{4\pi^2 m_{\chi 0} m_{\lambda 0}}{g_0^4} \right) u + \text{h.c.} \right] + \\ + |\Lambda|^2 \left| \frac{4\pi^2 m_{\lambda 0}}{g_0^2} \right|^2 \frac{|k_u|^2}{k_{u^\dagger u}} - \left(\Lambda^2 \frac{4\pi^2 m_{\chi 0} m_{\lambda 0}}{g_0^4} \frac{k_{u^\dagger}}{k_{u^\dagger u}} + \text{h.c.} \right) + \frac{|\Lambda|^2 |m_{\chi 0}|^2}{k_{u^\dagger u}}, \tag{4.30}$$

where $k_u = \partial k / \partial u$, etc. Here

$$\Lambda = \mu e^{-4\pi^2/g^2(\mu)} e^{i\Theta/4} \tag{4.31}$$

is the holomorphic scale in the $\mathcal{N} = 2$ limit. One can check that the dependence on the vacuum angle Θ and the fermion masses is the correct one dictated by the chiral anomaly. The result above does not have manifest $\mathrm{SU}(2)_R$ symmetry because the coefficients of $|m_{\lambda 0}|^2$ and $|m_{\chi 0}|^2$ are different. (Note that $m_{\lambda 0} m_{\chi 0} = \det(m_{\Psi 0})$ is $\mathrm{SU}(2)_R$ invariant.) The potential is $\mathrm{SU}(2)_R$ invariant if and only if

$$k k_{u^\dagger u} - |k_u|^2 = -\frac{1}{(4\pi^2)^2}\,. \tag{4.32}$$

This relation is in fact satisfied, as can be seen by differentiating the relation eq. (4.14). Since our formalism is not manifestly $\mathrm{SU}(2)_R$ invariant, this is a non-trivial check. Using eq. (4.32) we can simplify the expression for the potential to obtain

$$V = \frac{\Lambda^2}{4\pi^2} \sum_{j=1}^{6} m_{\text{soft},j}^2 f_j(u^\dagger, u)\,, \tag{4.33}$$

where

$$
\begin{aligned}
m^2_{\text{soft},1} &= m^2_{\phi 0}\,, & f_1 &= 4\pi^2 k\,, \\
m^2_{\text{soft},2} &= 4\pi^2 T_0\,, & f_2 &= \frac{1}{4\pi^2 k_{u^\dagger u}}\,, \\
m^2_{\text{soft},3} &= 4\pi^2\,\mathrm{Re}\,(\Delta_0)\,, & f_3 &= 8\pi^2\,\mathrm{Re}\left(2u - \frac{k_{u^\dagger}}{k_{u^\dagger u}}\right), \\
m^2_{\text{soft},4} &= 4\pi^2\,\mathrm{Im}\,(\Delta_0)\,, & f_4 &= -8\pi^2\,\mathrm{Im}\left(2u - \frac{k_{u^\dagger}}{k_{u^\dagger u}}\right), \\
m^2_{\text{soft},5} &= 4\pi^2\,\mathrm{Re}\left(\frac{m^2_{B0}}{g_0^2}\right), & f_5 &= 2\,\mathrm{Re}(u)\,, \\
m^2_{\text{soft},6} &= 4\pi^2\,\mathrm{Im}\left(\frac{m^2_{B0}}{g_0^2}\right). & f_6 &= -2\,\mathrm{Im}(u)\,,
\end{aligned}
\tag{4.34}
$$

where

$$
T_0 \equiv \frac{1}{g_0^4}\,\mathrm{tr}\,|m_{\Psi 0}|^2\,, \qquad \Delta_0 \equiv \frac{1}{g_0^4}\det(m_{\Psi 0})\,. \tag{4.35}
$$

These results agree with the results of the previous subsection obtained using $\mathcal{N}=2$ arguments.[9] We have factored out powers of $4\pi^2 = 16\pi^2/b$ according to the expectations of naïve dimensional analysis [27]. The overall factor of $1/(4\pi^2)$ arises because the potential is a 1-loop effect, while the factors of $4\pi^2$ in the definitions of $m^2_{\text{soft},j}$ are chosen so that these quantities are equal to soft parameters renormalized at the scale Λ where the theory becomes strong: $g^2(\Lambda) \sim 4\pi^2$. If naïve dimensional analysis is reliable, then the functions f_j should all be order 1.

The functions $f_1, \ldots, f_4$ are plotted in figures. 1–4. There are several interesting points to note about the results. First, note that the functions $f_1, \ldots, f_4$ are all of order 1, as predicted by naïve dimensional analysis. This is striking evidence for the correctness of these ideas in the context of supersymmetric theories.

Although the results we have derived are not justified near the monopole/dyon points, the behavior near these points is interesting. Note that $m^2_{\phi 0}$ and $\mathrm{Im}(\Delta_0)$ apparently drive the theory away from the monopole/dyon points, while T_0 drives the theory toward the monopole/dyon points. $\mathrm{Re}(\Delta_0)$ apparently gives a local minimum at either the monopole or dyon points, depending on the sign. When we will consider the theory near the monopole points, we will find that these conclusions are in fact correct.

Finally, note that the results above predict a rich phase structure as the various soft breaking terms are varied. To give only one example, it can be seen that there is a first-order phase transition between a Coulomb and a confined phase as we increase the ratio $m^2_{\Phi 0}/T_0$.

[9] As already mentioned in section 4.1, the soft terms induced by an $\mathcal{N}=2$ dilaton spurion and studied in ref. [7] correspond to the choice $m^2_{\text{soft},2} = -2m^2_{\text{soft},3}$ with all other soft terms vanishing. The $\mathcal{N}=1$ mass perturbation studied in ref. [3] simply corresponds to $m^2_{\text{soft},2} \neq 0$ with all other terms vanishing.

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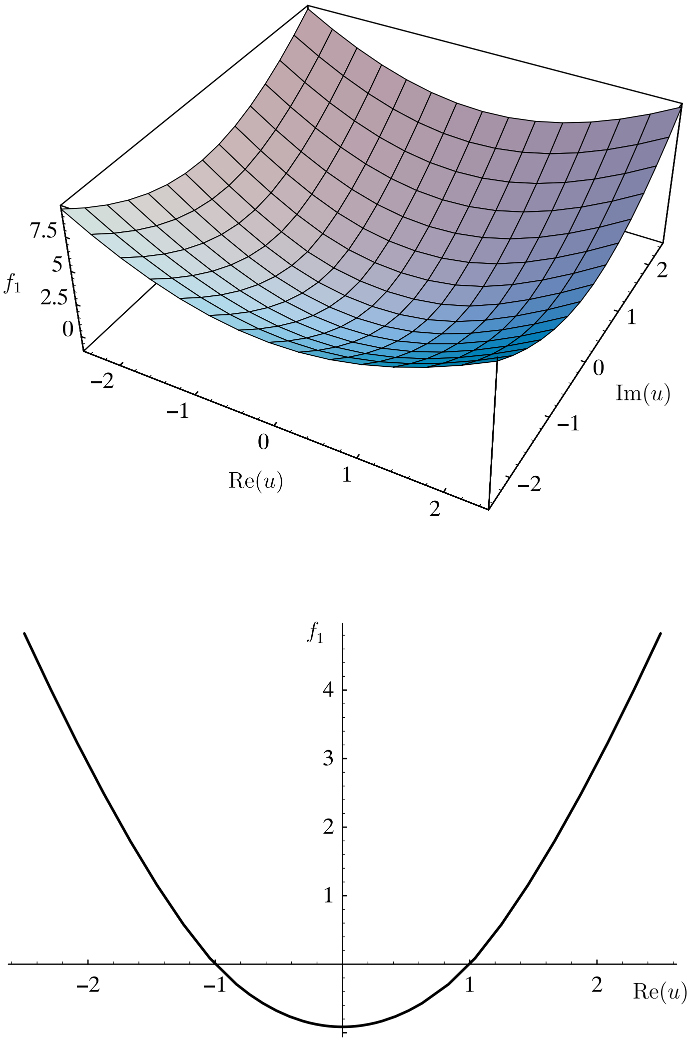

Figure 1: Potential induced by a soft scalar mass $m_{\phi 0}^2$.

We now consider the question of how close we can get to the monopole points $u = \pm 1$ before the results above break down. The reason that the effective theory breaks down near the monopole points is that there are extra monopoles (or dyons) with mass

$$m_M \sim \Lambda[\langle u \rangle - (\pm 1)]\,. \tag{4.36}$$

The Seiberg-Witten solution away from the monopole points gives the *exact* effective lagrangean (up to higher derivative terms) with the monopole integrated out.[10] Therefore, as long as $m_M \gg m_{\rm soft}$, it is a good approximation to integrate out the monopoles. This means that the results above are valid as long as

$$|\langle u \rangle - (\pm 1)| \gg \frac{m_{\rm soft}}{\Lambda}\,. \tag{4.37}$$

The corrections are suppressed by powers of $m_{\rm soft}/m_M$ for $|\langle u \rangle - (\pm 1)| \lesssim 1$, and are

[10] Higher derivative terms affect the scalar potential at $\mathcal{O}(m_{\rm soft}^3)$, while the leading effects we compute are order $m_{\rm soft}^2$. Therefore, higher derivative terms are negligible for small $m_{\rm soft}$.

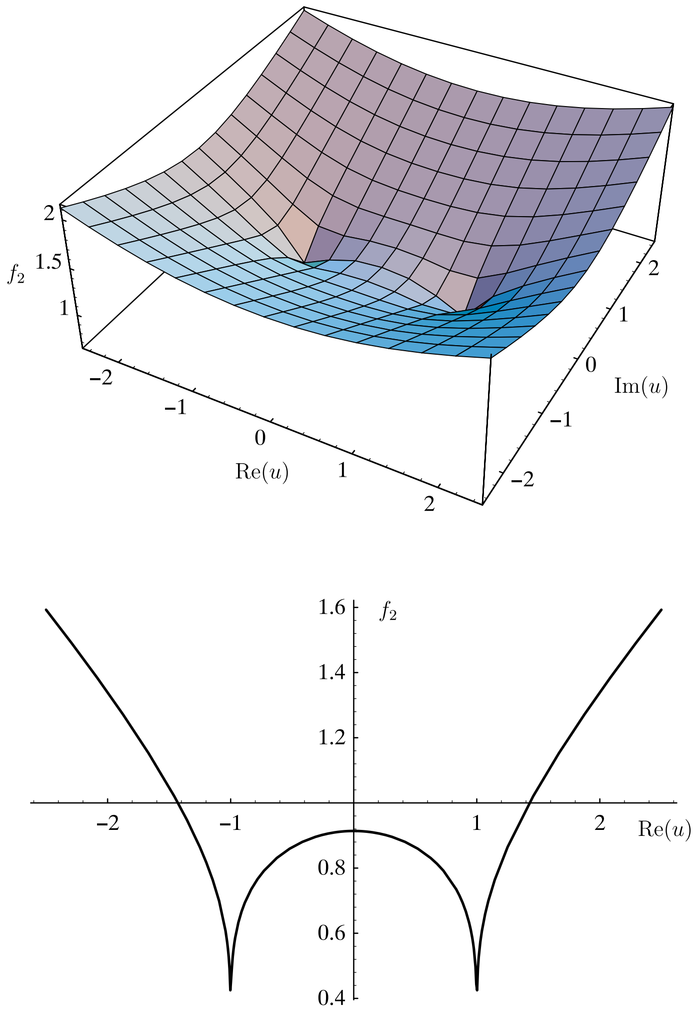

Figure 2: Potential induced by the trace of the fermion mass matrix T_0. The potential approaches a finite value at the cusp singularities at the monopole/dyon points $u = \pm 1$.

of order $m_{\rm soft}/\Lambda$ for $|\langle u\rangle - (\pm 1)| \gtrsim 1$. For $m_{\rm soft} \ll \Lambda$ this means that we can trust the above results up to a small region $|u-1| \lesssim m_{\rm soft}/\Lambda$. Inside this region the monopole VEV's can be turned on and decrease the energy. However we will later show that this effect is parametrically small and it does not significantly alter the picture of where the vacuum resides. We can therefore conclude that $m^2_{\phi 0}$ and m^2_{B0} push the vacuum away from the monopole points, while fermion masses $m_{\lambda 0}$ and $m_{\chi 0}$ tend to stabilize the monopole points.

4.4 Near the monopole points

We now describe the effective theory near the monopole point $\langle u\rangle = +1$.[11] As argued in ref. [3], near $\langle u\rangle = +1$ the weakly-coupled light degrees of freedom are the modulus field u, the *dual* photon field v_D, and the monopole fields M, $\bar{M}$. The

[11] The point $\langle u\rangle = -1$ is related to this point by charge conjugation.

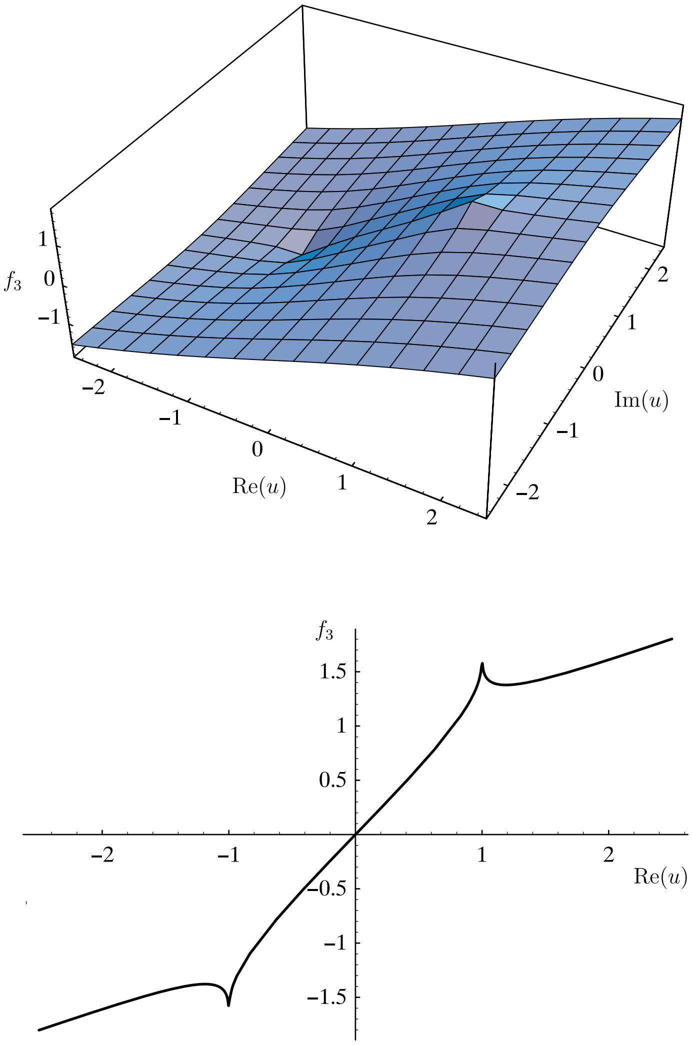

Figure 3: Potential induced by Re(Δ_0), where Δ_0 is the determinant of the fermion mass matrix. The potential approaches a finite value at the cusp singularities at the monopole/dyon points $u = \pm 1$.

effective lagrangean is therefore

$$\begin{aligned}
\mathcal{L}_{\text{eff}} = & \int d^2\theta d^2\bar{\theta}\, \Lambda_R^2 k_D(u^\dagger, u) + \left(\int d^2\theta\, \frac{1}{2} s_D(u) w_D^\alpha w_{D\alpha} + \text{h.c.} \right) + \\
& + \int d^2\theta d^2\bar{\theta}\, \mathcal{Z}_M(u^\dagger, u) \left(M^\dagger e^{v_D} M + \bar{M}^\dagger e^{-v_D} \bar{M} \right) + \\
& + \left[\int d^2\theta \left(\sqrt{2} \bar{M} a_D(u) M + \Lambda_S^2 m u \right) + \text{h.c.} \right] + \\
& + O(M^4) + \text{derivative terms}\,.
\end{aligned} \tag{4.38}$$

A term of the form [13]

$$\int d^2\theta\, w_D^\alpha \bar{D}^2 D_\alpha \ln \hat{\mathcal{Z}}_\Phi + \text{h.c.} \tag{4.39}$$

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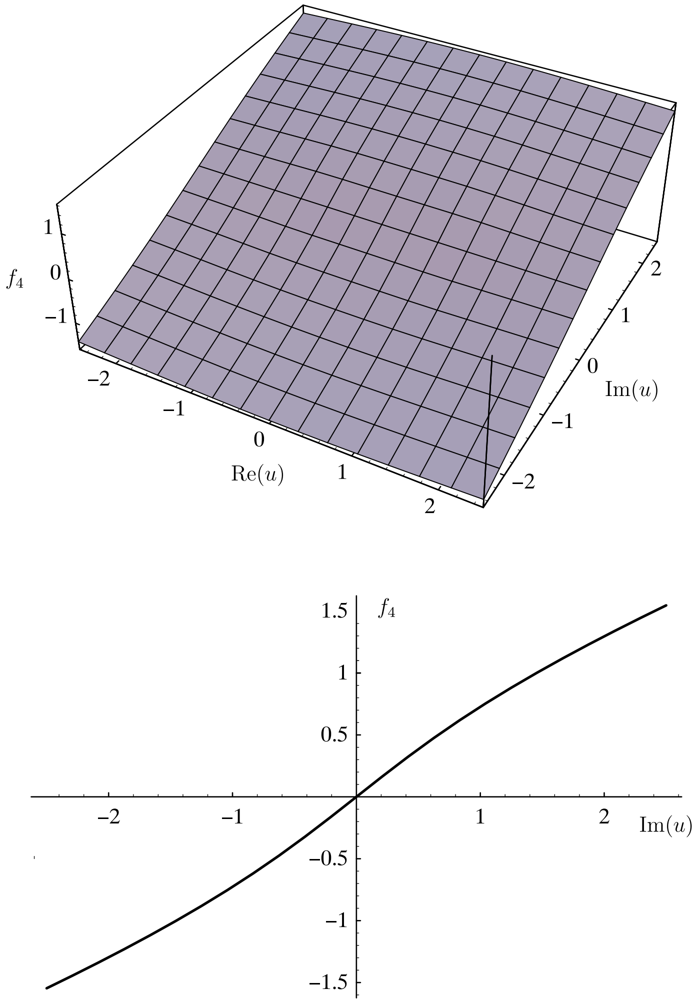

Figure 4: Potential induced by $\mathrm{Im}(\Delta_0)$, where Δ_0 is the determinant of the fermion mass matrix.

is absent by charge conjugation. (A careful discussion of charge conjugation is given below.) The monopole fields form a $\mathrm{SU}(2)_R$ doublet

$$\mathcal{M}^j = \begin{pmatrix} M \\ i\bar{M}^\dagger \end{pmatrix}. \tag{4.40}$$

Note that M is in a doublet with $\bar{M}^\dagger$ (rather than $\bar{M}$) because the doublet must have well-defined U(1) gauge charge. The factor of i is required by $\mathrm{SU}(2)_R$: the coefficient of the term $\psi_{\bar{M}} a_D \psi_M$ is real, while the coefficient of the terms $\psi_{\bar{M}}\psi M$ and $\psi_{\bar{M}}\lambda M$ are both imaginary.

The Seiberg-Witten prepotential gives the exact effective Kähler potential and gauge coupling with the monopoles integrated out. Since the effective theory eq. (4.38) includes the monopole fields, we must 'integrate in' the monopoles, i.e. invert the process of integrating out the monopoles. In a general theory this is not unique, but in the present case we need the effective Kähler potential and gauge coupling only in the $\mathcal{N}=2$ limit, where the result of integrating out the monopoles is exhausted by a 1-loop calculation. We therefore have

$$s_D(\mu) = s_D^{(\mathrm{SW})} + \frac{1}{8\pi^2}\left(\ln\frac{a_D}{\mu} + 1 + c\right), \tag{4.41}$$

$$k_D(\mu) = k^{(\mathrm{SW})} + \frac{a_D^\dagger a_D}{8\pi^2}\left(\ln\frac{a_D^\dagger a_D}{\mu^2} + c\right), \tag{4.42}$$

where 'SW' denotes the Seiberg-Witten solution with the monopoles integrated out, and c is a scheme-dependent constant. (The analytic corrections to s_D and k are related by $\mathcal{N}=2$ SUSY.) We will set $c=0$ from now on. Here μ is a renormalization scale that is required because the effective theory containing the monopoles has marginal interactions, and hence logarithmic renormalization effects. As a consistency check, we note that s_D and the Kähler metric $(k_D)_{u^\dagger u}$ derived from eq. (4.41) are non-singular as $u \to 1$ ($a_D \to 0$).

In order to determine $\mathcal{Z}_M$ in eq. (4.38), we must discuss the transformation of the monopole fields under the anomalous U(1) transformation eq. (4.28). Note that the anomalous U(1) is broken both explicitly (by anomalies) and spontaneously (by $\langle u\rangle \neq 0$). Furthermore, the monopole fields are not in any sense simple functions of UV fields, so we must proceed carefully. The most general transformation law allowed by holomorphy, U(1) gauge invariance, and dimensional analysis is

$$M \mapsto f\left(A, u, \frac{\bar{M}M}{\Lambda_S^2}\right)\cdot M, \quad \bar{M} \mapsto \bar{f}\left(A, u, \frac{\bar{M}M}{\Lambda_S^2}\right)\cdot \bar{M}, \tag{4.43}$$

where A is the anomalous U(1) transformation parameter. The explicit (anomalous) breaking of $\mathrm{U}(1)_A$ is contained entirely in the fact that Λ_S is not invariant. The monopole term in the superpotential is therefore $\mathrm{U}(1)_A$ invariant, which gives

$$\bar{f}f = e^{-A}. \tag{4.44}$$

(Recall that $a_D \propto \Lambda_S$, so a_D has charge $+1$.) Finally, charge conjugation (defined below) exchanges M and $\bar{M}$, and therefore implies $\bar{f} \equiv f$. We conclude that the monopole and antimonopole transform linearly under the anomalous U(1) with the same charge $-1/2$. (The details of the charge conjugation argument are as follows. Define C in the ultraviolet theory as

$$C:\ V \mapsto -V^T, \qquad \Phi \mapsto +\Phi^T. \tag{4.45}$$

This is a symmetry of the UV lagrangean, and the positive sign for Φ is chosen so that $\langle\Phi\rangle \neq 0$ does not break C. (In a manifestly $\mathcal{N}=2$ symmetric description, C is therefore an R symmetry.) The coupling spurions $\mathcal{Z}_\Phi$ and S are clearly invariant under C. When the SU(2) gauge group breaks to U(1), the fields in the effective theory transform as

$$C: \ v \mapsto -v\,, \qquad u \mapsto u\,, \tag{4.46}$$

where v is the U(1) gauge superfield. This is obvious far from the origin where the theory is weakly coupled, and cannot change in the strong-coupling region because of continuity. Because the monopole fields have opposite charge under the 'dual' U(1) gauge group, they must transform as

$$C: \ M \leftrightarrow \pm\bar{M} \tag{4.47}$$

under C, implying $\bar{f} \equiv \pm f$. The negative sign is ruled out by the fact that $f = +1$ for the identity transformation $A = 0$.)

Now that we know how the monopole fields transform under the anomalous U(1) transformation eq. (4.28), we can fix $\mathcal{Z}_M$ in eq. (4.38) as a function of the UV couplings. $\mathcal{Z}_M$ does not run by $\mathcal{N} = 2$ SUSY, so it must be an RG-invariant function of $\hat{\mathcal{Z}}_\Phi$ and Λ. It cannot depend on Λ by dimensional analysis, and the anomalous U(1) tells us

$$\mathcal{Z}_M = c\hat{\mathcal{Z}}_\Phi^{-1/2}, \tag{4.48}$$

where c is a constant that is fixed by the $\mathcal{N} = 2$ limit:

$$\mathcal{Z}_M|_{\theta=\bar{\theta}=0} = 1\,. \tag{4.49}$$

This result can also be obtained using the gauged non-anomalous $\mathrm{U}(1)_R$ described in section 3. One has $R_\Phi = 0$ so that $\hat{\mathcal{Z}}_\Phi = e^{-2V_R/3}$. On the other hand, by charge symmetry and R-invariance of the low-energy theory, the monopole R charges are $R_M = R_{\bar{M}} = 1$. Therefore $\mathcal{Z}_M = e^{V_R/3}$ consistent with eq. (4.48).

A simple but remarkable consequence of these results is that the monopole soft mass does not run to all orders in perturbation theory in the low-energy theory. This is *a priori* surprising because the theory has no unbroken SUSY and has marginal interactions. The reason is simply that the wavefunction parameter of the monopoles does not run in the $\mathcal{N} = 2$ limit. The running of the soft masses is obtained by analytically continuing the running in the SUSY limit into superspace [13], and is therefore controlled by the SUSY limit.

Straightforward calculation gives the potential to be

$$\begin{aligned} V = V_0 &+ (2|a_D|^2 - \frac{1}{2}m_{\phi 0}^2)(|M|^2 + |\bar{M}|^2) + \frac{g_D^2}{2}(|M|^2 + |\bar{M}|^2)^2 - \\ &- \frac{\sqrt{2}g_D^2\Lambda}{4\pi^2}\left(\frac{\partial a_D}{\partial u}\right)^{-1}\left[\frac{4\pi^2 m_{\lambda 0}}{g_0^2}\bar{M}M + \frac{4\pi^2 m_{\chi 0}}{g_0^2}(\bar{M}M)^\dagger\right] + \text{h.c.}\,, \end{aligned} \tag{4.50}$$

where V_0 is the potential given in eqs. (4.33) and (4.34) with the replacement $k \to k_D$, and the running dual gauge coupling is $1/g_D^2 \equiv [s_D]_0$ (see eq. (4.41)). Note that this potential is $\mathrm{SU}(2)_R$ invariant, since

$$\begin{aligned}\mathcal{M}_j^\dagger \mathcal{M}^j &= |M|^2 + |\bar{M}|^2\,,\\ \mathcal{M}_j^\dagger \epsilon^{jk} (m_{\Psi 0})_{k\ell} \mathcal{M}^\ell &= -i\left(m_{\lambda 0}\bar{M}M + m_{\chi 0}(\bar{M}M)^\dagger\right).\end{aligned} \tag{4.51}$$

We now consider the energetics of the potential near the monopole points. For this purpose, it is convenient to expand the potential in powers of $u' = u - 1 \simeq 2ia_D$ and write

$$\begin{aligned}V &= g_D^2 \left| \sqrt{2}\bar{M}M + \frac{2i\Lambda}{g_0^2}(m_{\lambda 0}^* - m_{\chi 0}) \right|^2 + \frac{g_D^2}{2}(|M|^2 - |\bar{M}|^2)^2 + \\ &\quad + \left[-\frac{1}{2}m_{\phi 0}^2 + \frac{1}{2}|u'|^2\Lambda^2 + O(u'^4) \right] (|M|^2 + |\bar{M}|^2) V_{\text{Naive}}(u=1) + \\ &\quad + \Lambda^2 \left[\left(m_{\phi 0}^2 k_u(u=1) + 8\pi^2\Delta_0 + \frac{m_{B0}^2}{g_0^2} \right) u' + \text{h.c.} \right] + \mathcal{O}(m_{\text{soft}}^2 u'^2)\,,\end{aligned} \tag{4.52}$$

where

$$V_{\text{Naive}}(u=1) = V_0(u=1) + 4g_D^2\Lambda^2 \left| \frac{m_{\lambda 0} - m_{\chi 0}^*}{g_0^2} \right|^2 \tag{4.53}$$

is the potential with the monopole fields set to zero. The above form of the potential allows us to easily understand the origin of the cusp singularities in figures 2 and 3. These arise if we set $M = \bar{M} = 0$ and evaluate the running coupling $g_D^2(\mu)$ at a renormalization scale equal to the supersymmetric monopole mass of order $|a_D|$. Since $g_D^2(\mu) \sim 1/\ln\mu$ for $\mu \ll \Lambda$, this gives a logarithmic singularity as $a_D \to 0$. This singularity is smoothed out when we minimize the full potential because the monopole masses do not go to zero as $a_D \to 0$ in the presence of soft SUSY breaking. (The quantum corrections to the effective potential are well approximated by evaluating the running coupling g_D^2 at a renormalization scale of order the monopole VEV.)

We now turn to the monopole VEV's. Assuming that $m_{\lambda 0} - m_{\chi 0}^*$ is nonzero and all soft masses are of the same order, the monopole VEV's are essentially determined by minimizing the first two terms as long as $|u'|^2 \ll |m_{\lambda 0} - m_{\chi 0}^*|/\Lambda$. This gives

$$|\langle M \rangle|^2 \simeq |\langle \bar{M} \rangle|^2 \simeq \sqrt{2}\Lambda \left| \frac{m_{\lambda 0}^* - m_{\chi 0}}{g_0^2} \right|. \tag{4.54}$$

Note that this justifies dropping the $\mathcal{O}(M^4)$ terms in the effective lagrangean eq. (4.38), since they contribute to the vacuum energy at most $m_{\text{soft}}^2 \langle M \rangle^4 \sim m_{\text{soft}}^4$.[12] The perturbation $m_{\lambda 0}^* - m_{\chi 0}$ is equivalent to an $\mathcal{N} = 1$ superpotential mass, so the system is close to the confining phase found in ref. [3].

[12]Higher order terms in the monopole fields are severely constrained because the monopole fields are short $\mathcal{N} = 2$ multiplets. However, it is interesting to note that we do not need the power of $\mathcal{N} = 2$ SUSY to justify dropping these terms.

The monopole VEVs induce a positive mass-squared for u' of order $\Lambda|m_{\lambda 0}^* - m_{\chi 0}|$. This is larger than the $\mathcal{O}(m_{\rm soft}^2)$ contributions neglected in eq. (4.52), so this stabilizes the modulus at $u' \sim m_{\rm soft}$. Therefore the modulus is near the monopole points, and the approximations made above are consistent.

We now consider the effect of the monopole VEVs on the vacuum energy. This is important for determining whether there are first order phase transitions between the monopole points and other local minima on the moduli space. By the above qualitative discussion, it is easy to conclude that the value of the potential at the minimum near the monopole points is $V_{\rm Naive}(u = 1) + \mathcal{O}(\Lambda m_{\rm soft}^3)$. To obtain this result, note that the terms in the first line of eq. (4.52) respect $\mathcal{N} = 1$ SUSY, and therefore almost cancel at the minimum. Their contribution is therefore only $\mathcal{O}(m_{\rm soft}^3)$ instead of $\mathcal{O}(m_{\rm soft}^2)$.

This result shows that the vacuum energy near the monopole points is well approximated by the value at the cusp singularities in the naïve potential given in figures 1–4 that neglects the monopoles. Therefore, we can use figures 1–4 to decide if the vacuum is in the monopole (or dyon) region.[13] At these points there are always at least local minima with

$$\langle M\rangle^2, \langle D\rangle^2 \simeq \sqrt{2}\Lambda \left| \frac{m_{\lambda 0} \mp m_{\chi 0}^*}{g_0^2} \right| = \frac{\sqrt{2}\Lambda}{4\pi^2} \left(m_{\rm soft,2}^2 \mp 2 m_{\rm soft,3}^2 \right)^{1/2}. \tag{4.55}$$

When $m_{\rm soft,1}^2$, $m_{\rm soft,5}^2$, $m_{\rm soft,6}^2$ are sufficiently smaller than the other soft terms, we have that for $m_{\rm soft,3}^2 < 0$ the global minimum is at the monopole point, while for $m_{\rm soft,3}^2 > 0$ it is at the dyon point. This can be seen from figures 2 and 3. Notice also that in the limiting cases $m_{\rm soft,2}^2 = \pm 2 m_{\rm soft,3}^2$ the local monopole (dyon) VEV disappears [7]. On the other hand, when $m_{\rm soft,1}^2$ is sufficiently larger than the other soft terms the local minimum at the origin $u = 0$ is the global minimum. Indeed, when $m_{\rm soft,1}^2$ dominates, the monopole vevs are even less important to the vacuum energetics. As can be inferred from eq. (4.52), they can decrease the energy only for $|u'|^2 < m_{\phi 0}^2/\Lambda^2$ and only by an amount $\Delta V \sim -m_{\phi 0}^4/g_D^2$.

5. Conclusions

We have considered the most general soft SUSY breaking of $\mathcal{N} = 1$ and $\mathcal{N} = 2$ theories, including non-holomorphic perturbations. Using the method of ref. [8] we are able to obtain exact results when the soft masses are small compared to the scale of non-perturbative physics ($m_{\rm soft} \ll \Lambda$) because SUSY relates soft mass terms to background gauge fields. We gave a new formulation of this correspondence in terms of a non-anomalous gauged $\mathrm{U}(1)_R$ symmetry in a supergravity background. We also

[13] In figures 1–4 we plot $V/m_{\rm soft}^2$; since $\partial^2 V/\partial u^2 = \mathcal{O}(m_{\rm soft})$ at the monopole minimum, the second derivative is large in this plot.

applied this formalism to several cases of interest: $\mathcal{N}=1$ theories deformed moduli spaces and conformal fixed points, and $\mathcal{N}=2$ super-Yang-Mills.

Our results show that in many cases, the theory for $m_{\text{soft}} \ll \Lambda$ is in a different phase than the $m_{\text{soft}} \to \infty$ limit. For example, in the $\mathcal{N}=2$ SU(2) super-Yang-Mills theory, adding a small soft scalar mass drives the theory to a free Coulomb phase, while we believe that the $m_{\text{soft}} \to \infty$ theory is in a confining phase. This means that there are necessarily phase transitions as a function of the soft masses at $m_{\text{soft}} \sim \Lambda$. For example, this is important for non-perturbative studies of these models on the lattice, where supersymmetry presumably has to be imposed by tuning lattice parameters. Clearly, the road to understanding the relationship between supersymmetric and non-supersymmetric gauge theories remains a long one, but we hope that the steps taken in this paper will prove useful.

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